

Agora de la Gestion Financière

« Optimal Long-Term Allocation
for a Defined-Contribution Pension Fund »

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Pension Funds

- Defined Benefits (DB; Primauté de Prestations)
 - Your benefit is 60% of last wage
 - Risk is with employer
- Defined Contributions (DC; Primauté de Cotisations)
 - We manage your savings for retirement the best we can
 - Risk is with employee (Switzerland is in between)
- Changing demographics, and financial crisis

⇒ Shift from DB to DC

Contributions

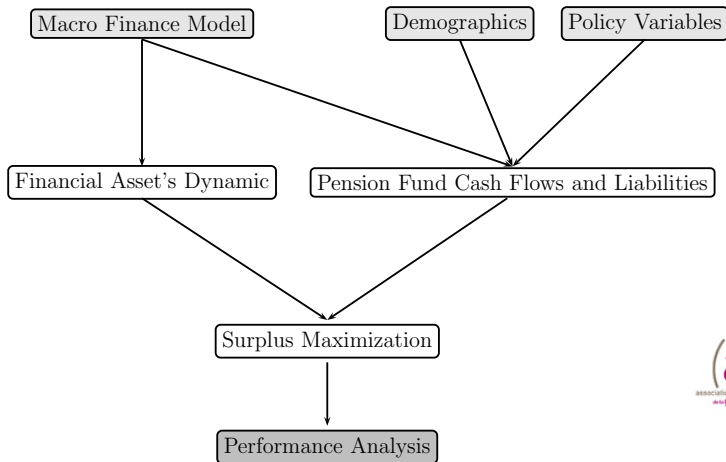
- We allocate 9 asset classes for a DC pension fund portfolio.
- Calibration: quarterly between 1985:Q1 and 2013:Q2
- Geography: USA, the Euro Area, Switzerland + Exchange rates
- Main results:
 - It is important to take liabilities into account
 - Bonds with 10 to 20 years are not sufficient for liability hedging
 - Discounting is complex: Macro forecast better than historical rates projections
 - Short sale constraints reduce performance

Organization of Presentation

- Economic and financial model
- Calibrate a pension fund
- Generate future liabilities and associate returns
- Seek portfolio allocation that maximizes surplus

Macro-Factors and Asset-Liability Correlation

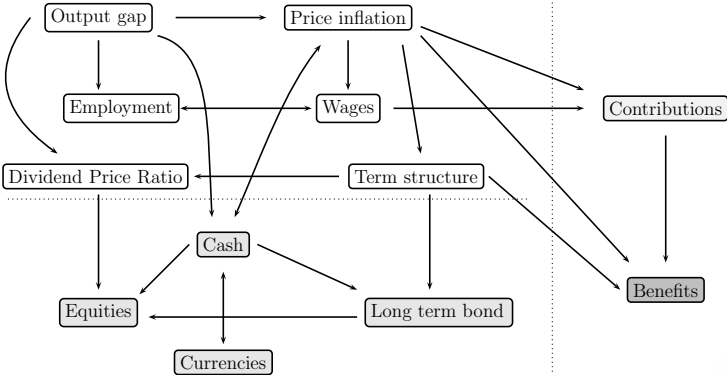
The Logic



Macro-Economics and Finance

Macroeconomic and Financial Factors

Pension Fund Liabilities



Financial Assets



Flavour of Equations: Short-Term Interest Rates

- Modeling approach: restricted VAR

$$A_0 \Delta X_{t+1} = \tilde{\mu} + A_1 \Delta X_t + A_2 X_t + A_3 \varepsilon_{t+1}$$

- Short-term interest rates $r_{t+1}^{(3m)}$ relate with output gap og_{t+1} , and inflation π_{t+1}

$$r_{t+1}^{(3m)} = \mu_{r,0} + \mu_{r,1} \pi_{t+1} + \mu_{r,2} og_{t+1} + \varepsilon_{r,t+1}$$

The short-run dynamics yields the driving factor $\hat{\varepsilon}$ of the long-term, error-correction mechanism

$$\begin{aligned} \Delta r_{t+1}^{(3m)} &= \mu_{\Delta r,0} + \mu_{\Delta r,1} \hat{\varepsilon}_{r,t} + \mu_{\Delta r,2} \Delta \pi_{t+1} \\ &+ \mu_{\Delta r,3} \Delta og_{t+1} + \varepsilon_{\Delta r,t+1} \end{aligned}$$

Flavor of Equations: Term-Structure of Interest Rates

- Economic Approach:

- Model 3M T-Bills, 2 year, and 10 year government bond rates
- Fit Nelson-Siegel and recover entire structure
- Discount using $R_{t+T}^{(i)} = R_{t+T}^{(i),Gov} + \pi$. $\pi = 1\%$ or 2%

- Regulatory Approach

- 'technical rate' : average of long-term government bond rate and the return of a risky portfolio, smoothed over a long period of time.
 - Danger of lagging behind

Liability Side : Population Dynamic

- Approach based on Markov chain:
 - Individual alive this year, next year will be:
 - one year older and alive
 - disabled and one year older
 - dead
 - Individual disabled this year, may next year be:
 - one year older, alive and well
 - disabled and one year older
 - dead
- Framework adaptable for longevity investigations
- Open pension fund: ψ (= 1% then replace actives)

Liabilities L_{t+T}

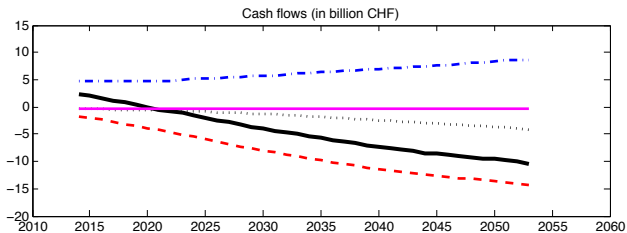
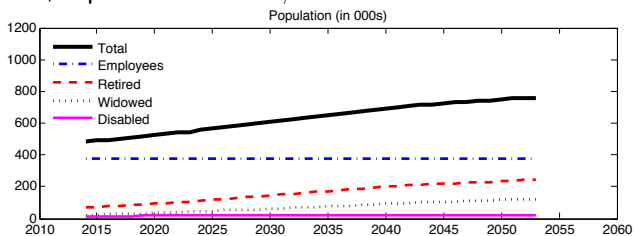
- L_{t+T} is the present value of expected future cash flows CF_{t+T+i}

$$L_{t+T} = E_t \sum_{i=1}^{\infty} \frac{CF_{t+T+i}}{(1 + R_{t+T}^{(i)})^i} \quad CF_t = \text{Pensions} - \text{Contributions}$$

- Return on Liabilities $r_{L,t+1} = \log(L_{t+1}/L_t)$

Population and Cash-Flow Dynamic

- Scenario 1, replacement rate $\psi = 1$



- CF prospective is not looking very good

Duration of Liabilities

- Duration of liabilities:

$$D_t = \frac{1}{L_t} E_t \sum_{i=1}^{\infty} i \frac{CF_{t+i}}{(1 + R_t^{(i)})^i} \quad \text{where} \quad L_t = E_t \sum_{i=1}^{\infty} \frac{CF_{t+i}}{(1 + R_t^{(i)})^i}.$$

Premium	$\psi = 0.8$	$\psi = 1.0$	$\psi = 1.2$
$\pi = 1\%$	50.9	70.1	92.4
$\pi = 2\%$	41.6	52.9	66.8

- Conclusion: 10 year bonds do not have the right duration. Need longer-term instruments

Building Intuition with Gordon-Shapiro for Next Steps



$$L_t = \frac{CF}{R_t}$$

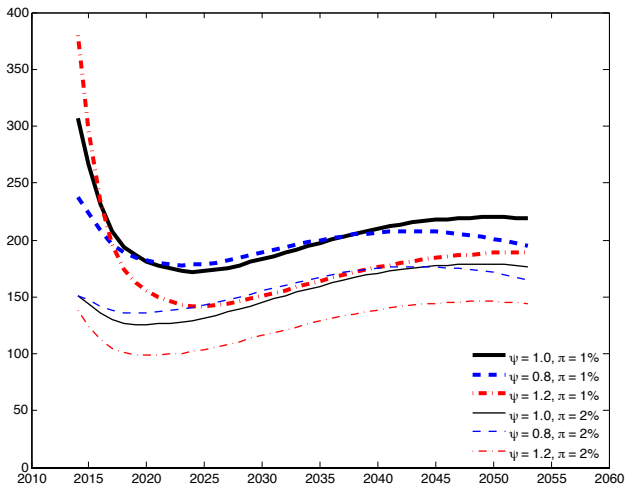
increase discount rate from 2.5% to 4%, liability decrease by 37%.



$$L_t = \frac{CF}{R_t - g}$$

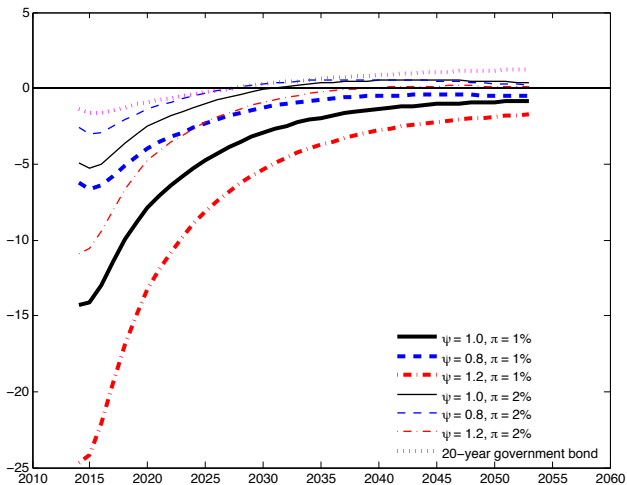
take $g = 1\%$. Same increase of R_t , liability decrease by 50%.

Evolution of Liabilities



- In short run, huge uncertainty. What is my liability?

Return on Liabilities



- Intuition: If $CF_t = C, L_t = C/R_t$ and $r_{L,t+1} = \log(L_{t+1}/L_t) = \log(R_t) - \log(R_{t+1})$
- Expect large negative liability returns in short turn.

Real Expected Return and Volatility of Assets (Achtung: 20 years Horizon)

	Expected return	Volatility
Cash	0.34	3.05
U.S. bond	1.31	4.19
U.S. equity	7.87	9.43
E.A. bond	0.67	5.06
E.A. equity	6.50	11.08
Swiss bond	0.49	4.23
Swiss equity	9.35	11.70
Commodities	6.61	23.12
Swiss real estate	3.31	6.11

Hedging Properties of Asset Classes

(in %)	Correlation with inflation	Correlation with liabilities	
		$\psi = 1.0$ $\pi = 1\%$	$\psi = 1.0$ $\pi = 2\%$
Assets	(nominal)	(real)	(real)
Cash	79.5	-40.2	-51.9
U.S. bond	44.4	50.5	58.5
U.S. equity	28.8	29.9	34.5
E.A. bond	25.3	54.4	61.5
E.A. equity	10.7	23.2	22.9
Swiss bond	26.2	58.3	70.3
Swiss equity	11.8	19.5	21.8
Commodities	64.8	-23.4	-22.1
Swiss real estate	25.4	27.9	37.0
Inflation	—	(nom.) -18.5	(nom.) -24.6

Optimal Assets-Only and Assets-Liabilities Portfolios

20 years horizon, surplus maximization

(in %) ($\psi = 1$ and $\pi = 1\%$)	Assets-Only			Assets-Liabilities		
	GMVP	$\lambda = 50$	$\lambda = 20$	LHP	$\lambda = 50$	$\lambda = 20$
No weight restriction						
Cash	53.0	19.5	-30.7	-129.0	-158.8	-203.5
U.S. bond	-17.9	10.3	52.6	55.3	82.1	122.2
U.S. equity	11.1	18.3	29.0	9.7	16.9	27.7
E.A. bond	14.9	6.5	-6.0	44.2	35.3	21.9
E.A. equity	-2.0	-1.1	0.2	3.0	3.8	4.9
Swiss bond	36.4	14.3	-18.8	133.0	108.9	72.9
Swiss equity	-3.3	9.8	29.3	8.3	21.1	40.3
Commodities	2.7	7.2	14.0	-0.2	4.4	11.3
Swiss real estate	5.0	15.2	30.3	-24.3	-13.6	2.5
μ_A	1.1	3.9	8.0	2.0	4.8	8.8
σ_A	2.1	3.2	6.3	7.9	8.4	10.2
μ_S	3.4	6.2	10.2	4.3	7.0	11.1
σ_S	13.6	13.6	14.4	11.2	11.4	12.7
Cost of Assets-Only allocation	-	-	-	29.2	14.3	5.5
Cost of positivity restrictions	0.2	0.0	0.1	13.2	7.7	4.2
Positivity restrictions						
μ_A	1.4	3.9	7.0	1.3	3.3	6.3
σ_A	2.2	3.2	5.6	3.4	3.9	6.0
μ_S	3.7	6.2	9.3	3.6	5.6	8.6
σ_S	13.5	13.6	14.3	12.3	12.5	13.3
Cost of Assets-Only allocation	-	-	-	14.9	6.5	2.1

Merci pour votre attention !

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